B.A. /B.Sc. Part-III (Honours) Examination, 2020 (1+1+1)
Subject: Mathematics
Paper: VI

Time: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer *any three* questions: 3×10 = 30

(a) From D'Alembert's principle, obtain the general equation of motion of a rigid body. Prove that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia was fixed and the same forces acted on the body. 5+5

(b) Forces X, Y, Z act along the three straight lines $y = b, z = -c$; $z = c, x = -a$; $x = a$, $y = -b$ respectively. Show that they will have a single resultant if $\frac{a}{X}+\frac{b}{Y}+\frac{c}{Z}=0$ and that the equations of its line of action are given by any two of the following three $\frac{y}{Y}-\frac{z}{Z}-\frac{a}{X}=0$, $\frac{z}{Z}-\frac{x}{X}-\frac{b}{Y}=0$ and $\frac{x}{X}-\frac{y}{Y}-\frac{c}{Z}=0$. 10

(c) A sphere of radius a, is projected up an inclined plane with velocity V and angular velocity ω in the sense which would cause it to roll up. If $V > a\omega$ and the coefficient of friction μ exceeds $\frac{2}{7}\tan\alpha$, show that the sphere will cease to ascend after time $\frac{5V+2a\omega}{5g\sin\alpha}$, where α is the inclination of the plane to the horizontal. 10

(d) State and prove a necessary and sufficient condition for equilibrium of a fluid under the action of external forces. 10

(e) Define surfaces of equi-pressure and equi-density. Prove that in a liquid at rest under the action of a force towards a fixed point and varying as the distance, the surfaces of equal pressure are concentric sphere. 4+6

2. Answer *any four* questions: $4\times5 = 20$

(a) A rod of length $2a$ is suspended by a string of length l attached to one end; if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}\ .$$ 5

(b) Find the moments and products of inertia with respect to rectangular axis, parallel to the co-ordinate axes through the point $(a,0)$ on the circumference for the uniform circular disc $x^2+y^2 = a^2$ of mass M. Also write down the corresponding inertia matrix. 5

(c) A rough solid circular cylinder rolls down a second rough cylinder which is fixed with its axis horizontal. If the plane through their axis makes an angle α with the vertical when the first cylinder is at rest, show that the cylinders will separate when the angle of elevation is $\cos^{-1}\left(\frac{4}{7}\cos\alpha\right)$ 5

(d) Prove that for a catenary of uniform strength, the tension varies as the radius of curvature. 5

(e) A circular area of radius 'a' is immersed in a homogeneous liquid with its plane vertical and centre at a depth 'h'; find the depth of the centre of pressure. 5

(f) Assuming the atmosphere to be in convective equilibrium, deduce the relation $\frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma}\frac{z}{H}$ by considering gravity to be constant. 5